FYJC - MATHEMATICS & STATISTICS

PAPER - I

Ex 5.7

INVERSE CIRCULAR

TRIGONOMETRIC FN'S

$\sin^{-1}(\sin\theta) = \theta$

$\cos^{-1}(\cos\theta) = \theta$

$\tan^{-1}(\tan\theta) = \theta$

$\sin(\sin^{-1}x) = x$

$\cos(\cos^{-1}x) = x$

$\tan(\tan^{-1}x) = x$

$\text{cosec}^{-1}(1/x) = \sin^{-1}x$

$\sec^{-1}(1/x) = \cos^{-1}x$

$\cot^{-1}(1/x) = \tan^{-1}x$

$\sin^{-1}x + \cos^{-1}x = \pi/2$

$\tan^{-1}x + \cot^{-1}x = \pi/2$

$\sec^{-1}x + \text{cosec}^{-1}x = \pi/2$

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

HOWEVER if xy > 1 THEN

$$\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

Q SET - 1

01. $\tan^{-1}\left[\frac{2}{3}\right] + \tan^{-1}\left[\frac{1}{5}\right] = \frac{\pi}{4}$

02. $\tan^{-1}\left[\frac{7}{9}\right] + \tan^{-1}\left[\frac{1}{8}\right] = \frac{\pi}{4}$

03. $\tan^{-1}\left[\frac{3}{5}\right] + \tan^{-1}\left[\frac{1}{4}\right] = \frac{\pi}{4}$

04. $\cot^{-1}[8] + \cot^{-1}\left[\frac{9}{7}\right] = \frac{\pi}{4}$

05.

$$\tan^{-1}\left[\frac{1}{2}\right] + \tan^{-1}\left[\frac{1}{5}\right] + \tan^{-1}\left[\frac{1}{8}\right] = \frac{\pi}{4}$$

06.

$$\tan^{-1}\left[\frac{3}{4}\right] + \tan^{-1}\left[\frac{3}{5}\right] - \tan^{-1}\left[\frac{8}{19}\right] = \frac{\pi}{4}$$

07.
$$2\tan^{-1}\left[\frac{1}{3}\right] + \tan^{-1}\left[\frac{1}{4}\right] = \tan^{-1}\left[\frac{16}{13}\right]$$

08.

$$2\tan^{-1}\left[\frac{1}{8}\right] + \tan^{-1}\left[\frac{1}{7}\right] + 2\tan^{-1}\left[\frac{1}{5}\right] = \frac{\pi}{4}$$

Q SET - 2

01.
$$\cos^{-1}\left[\frac{4}{5}\right] + \cos^{-1}\left[\frac{12}{13}\right] = \cos^{-1}\left[\frac{33}{65}\right]$$

02.
$$\sin^{-1}\left[\frac{3}{5}\right] + \cos^{-1}\left[\frac{12}{13}\right] = \sin^{-1}\left[\frac{56}{65}\right]$$

03.
$$\sin^{-1}\left[\frac{3}{5}\right] + \sin^{-1}\left[\frac{8}{17}\right] = \sin^{-1}\left[\frac{77}{85}\right]$$

04. $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{5}{13}\right) = \cot^{-1}\left(\frac{119}{120}\right)$

05. $2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$

06. $\sin^{-1}\left(\frac{3}{\sqrt{34}}\right) + \cos^{-1}\left(\frac{4}{\sqrt{17}}\right) = \frac{\pi}{4}$

07. $\sin^{-1}\left(\frac{2}{\sqrt{13}}\right) + \cos^{-1}\left(\frac{5}{\sqrt{26}}\right) = \frac{\pi}{4}$

08. Prove

$$2\cot^{-1}\left(\frac{3}{2}\right) + \sec^{-1}\left(\frac{13}{12}\right) = \frac{\pi}{2}$$

09. Evaluate

$$\sin\left(\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{5}{12}\right)\right)$$

Q SET - 3

01. $\tan^{-1}\left[\frac{\sin 2x}{1 + \cos 2x}\right] = x$

02. $\cot^{-1}\left[\frac{\sin 2x}{1 - \cos 2x}\right] = x$

03. $\tan^{-1}\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} = x$

04. $\tan^{-1}[\operatorname{cosec} x - \cot x] = x/2$

05. $\tan^{-1}\sqrt{\frac{1 - \cos x}{1 + \cos x}} = x/2$

06. $\tan^{-1}\left[\frac{\cos x - \sin x}{\cos x + \sin x}\right] = \pi/4 - x$

07. $\cot^{-1}\sqrt{\frac{1 - \sin x}{1 + \sin x}} = \pi/4 + x/2$

08. $\tan^{-1}\left[\frac{\cos x}{1 - \sin x}\right] = \pi/4 + x/2$

09. $\tan^{-1}\left[\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right] = x$

Q SET - 4

01. $\cos^{-1}(4x^3 - 3x) = 3\cos^{-1}x$

02. $\cos{-1}\left[\frac{1 - x^2}{1 + x^2}\right] = 2\tan^{-1}x$

03. $\sin{-1}\left[\frac{1 - x^2}{1 + x^2}\right] = \frac{\pi}{2} - 2\tan^{-1}x$

04. $\tan^{-1}\left[\frac{3x - x^3}{1 + 3x^2}\right] = 3\tan^{-1}x$

05. $\tan^{-1}\left[\frac{x}{\sqrt{a^2 - x^2}}\right] = \sin^{-1}\left[\frac{x}{a}\right]$

06. $\tan^{-1}\sqrt{\frac{1 - x}{1 + x}} = \frac{1}{2}\cos^{-1}x$

07. $\tan^{-1}\left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}.}{\sqrt{1+x^2} - \sqrt{1-x^2}.}\right] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}(x^2)$

SOLUTION TO Q SET - 1

01. $\tan^{-1}\left(\frac{2}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \frac{\pi}{4}$

$= \tan^{-1}\left(\dfrac{\frac{2}{3} + \frac{1}{5}}{1 - \frac{2}{3} \cdot \frac{1}{5}}\right)$

$= \tan^{-1}\left(\dfrac{\frac{10 + 3}{15}}{\frac{15 - 2}{15}}\right)$

$= \tan^{-1}\left(\frac{13}{13}\right)$

$= \tan^{-1}(1)$

$= \pi/ 4$

02. $\tan^{-1}\left(\frac{7}{9}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

$= \tan^{-1}\left(\dfrac{\frac{7}{9} + \frac{1}{8}}{1 - \frac{7}{9} \cdot \frac{1}{8}}\right)$

$= \tan^{-1}\left(\dfrac{\frac{56 + 9}{72}}{\frac{72 - 7}{72}}\right)$

$= \tan^{-1}\left(\frac{65}{65}\right)$

$= \tan^{-1}(1)$

$= \pi/ 4$

03. $\tan^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{1}{4}\right) = \frac{\pi}{4}$

$= \tan^{-1}\left(\dfrac{\frac{3}{5} + \frac{1}{4}}{1 - \frac{3}{5} \cdot \frac{1}{4}}\right)$

$= \tan^{-1}\left(\dfrac{\frac{12 + 5}{20}}{\frac{20 - 3}{20}}\right)$

$= \tan^{-1}\left(\frac{17}{17}\right)$

$= \tan^{-1}(1)$

$= \pi/ 4$

04. $\cot^{-1}[\,8\,] + \cot^{-1}\left(\frac{9}{7}\right) = \frac{\pi}{4}$

$= \tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{7}{9}\right) = \frac{\pi}{4}$

$= \tan^{-1}\left(\dfrac{\frac{1}{8} + \frac{7}{9}}{1 - \frac{1}{8} \cdot \frac{7}{9}}\right)$

$= \tan^{-1}\left(\dfrac{\frac{9 + 56}{72}}{\frac{72 - 7}{72}}\right)$

$= \tan^{-1}\left(\frac{65}{65}\right)$

$= \tan^{-1}(1)$

$= \pi/ 4$

05.

$$\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$$

LHS

$$=\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{2}\cdot\frac{1}{5}}\right)+\tan^{-1}\left(\frac{1}{8}\right)$$

$$=\tan^{-1}\left(\frac{\frac{5+2}{10}}{\frac{10-1}{10}}\right)+\tan^{-1}\left(\frac{1}{8}\right)$$

$$=\tan^{-1}\left(\frac{7}{9}\right)+\tan^{-1}\left(\frac{1}{8}\right)$$

$$=\tan^{-1}\left(\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{9}\cdot\frac{1}{8}}\right)$$

$$=\tan^{-1}\left(\frac{\frac{56+9}{72}}{\frac{72-7}{72}}\right)$$

$$=\tan^{-1}\left(\frac{65}{65}\right)$$

$$=\tan^{-1}(1)$$

$$=\pi/4$$

06.

$$\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{3}{5}\right)-\tan^{-1}\left(\frac{8}{19}\right)=\frac{\pi}{4}$$

LHS

$$=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}\cdot\frac{3}{5}}\right)-\tan^{-1}\left(\frac{8}{19}\right)$$

$$=\tan^{-1}\left(\frac{\frac{15+12}{20}}{\frac{20-9}{10}}\right)-\tan^{-1}\left(\frac{8}{19}\right)$$

$$=\tan^{-1}\left(\frac{27}{11}\right)-\tan^{-1}\left(\frac{8}{19}\right)$$

$$=\tan^{-1}\left(\frac{\frac{27}{11}-\frac{8}{19}}{1+\frac{27}{11}\cdot\frac{8}{19}}\right)$$

$$=\tan^{-1}\left(\frac{\frac{513-88}{88}}{\frac{209+216}{88}}\right)$$

$$=\tan^{-1}\left(\frac{425}{425}\right)$$

$$=\tan^{-1}(1)$$

$$=\pi/4$$

07.

$$2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{4}\right)=\tan^{-1}\left(\frac{16}{13}\right)$$

LHS

$$=\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{4}\right)$$

$$=\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{3}}{1-\frac{1}{3}\cdot\frac{1}{3}}\right)+\tan^{-1}\left(\frac{1}{4}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3+3}{9}}{\frac{9-1}{9}}\right) + \tan^{-1}\left(\frac{1}{4}\right)$$

$$= \tan^{-1}\left(\frac{6}{8}\right) + \tan^{-1}\left(\frac{1}{4}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{4}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{4}}{1-\frac{3}{4}\cdot\frac{1}{4}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{12+4}{12}}{\frac{16-3}{72}}\right)$$

$$= \tan^{-1}\left(\frac{16}{13}\right)$$

08.

$$2\tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{7}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) = \frac{\pi}{4}$$

$$2\tan^{-1}\left(\frac{1}{8}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \frac{\pi}{4}$$

LHS

$$= 2\tan^{-1}\left(\frac{\frac{1}{8}+\frac{1}{5}}{1-\frac{1}{8}\cdot\frac{1}{5}}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= 2\tan^{-1}\left(\frac{\frac{5+8}{40}}{\frac{40-1}{40}}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= 2\tan^{-1}\left(\frac{13}{39}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= 2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{3}}{1-\frac{1}{3}\cdot\frac{1}{3}}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3+3}{9}}{\frac{9-1}{9}}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{6}{8}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\cdot\frac{1}{7}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{21+4}{28}}{\frac{28-3}{72}}\right)$$

$$= \tan^{-1}\left(\frac{25}{25}\right)$$

$$= \tan^{-1}(1) \quad = \pi / 4$$

SOLUTION TO Q SET - 2

01. $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$

STEP 1

$\cos^{-1}\left(\frac{4}{5}\right) = A$

$\frac{4}{5} = \cos A$

$\cos^2 A + \sin^2 A = 1$

$\frac{16}{25} + \sin^2 A = 1$

$\sin^2 A = 1 - \frac{16}{25}$

$\sin^2 A = \frac{9}{25}$

$\sin A = \frac{3}{5}$

$\tan A = \frac{\sin A}{\cos A}$

$\tan A = \frac{3}{4}$

$A = \tan^{-1}\left(\frac{3}{4}\right)$

HENCE $\cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$

STEP 2

$\cos^{-1}\left(\frac{12}{13}\right) = B$

$\frac{12}{13} = \cos B$

$\cos^2 B + \sin^2 B = 1$

$\frac{144}{169} + \sin^2 B = 1$

$\sin^2 B = 1 - \frac{144}{169}$

$\sin^2 B = \frac{25}{169}$

$\sin B = \frac{5}{13}$

$\tan B = \frac{\sin B}{\cos B}$

$\tan B = \frac{5}{12}$

$B = \tan^{-1}\left(\frac{5}{12}\right)$

HENCE $\cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\left(\frac{5}{12}\right)$

STEP 3

$\cos^{-1}\left(\frac{33}{65}\right) = C$

$\frac{33}{65} = \cos C$

$\cos^2 C + \sin^2 C = 1$

$\frac{33^2}{65^2} + \sin^2 C = 1$

$\sin^2 C = 1 - \frac{33^2}{65^2}$

$\sin^2 C = \frac{65^2 - 33^2}{65^2}$

$\sin^2 C = \frac{(65 + 33)(65 - 33)}{65^2}$

$\sin^2 C = \frac{(98)(32)}{65^2}$

$\sin^2 C = \frac{(49)(64)}{65^2}$

$\sin C = \dfrac{7 \times 8}{65}$

$\sin C = \dfrac{56}{65}$

$\tan C = \dfrac{\sin C}{\cos C}$

$\tan C = \dfrac{56}{33}$

$C = \tan^{-1}\left(\dfrac{56}{33}\right)$

HENCE $\cos^{-1}\left(\dfrac{33}{65}\right) = \tan^{-1}\left(\dfrac{56}{33}\right)$

STEP 4 :

$\cos^{-1}\left(\dfrac{4}{5}\right) + \cos^{-1}\left(\dfrac{12}{13}\right) = \cos^{-1}\left(\dfrac{33}{65}\right)$

WE PROVE

$\tan^{-1}\left(\dfrac{3}{4}\right) + \tan^{-1}\left(\dfrac{5}{12}\right) = \tan^{-1}\left(\dfrac{56}{33}\right)$

LHS

$= \tan^{-1}\left(\dfrac{3}{4}\right) + \tan^{-1}\left(\dfrac{5}{12}\right)$

$= \tan^{-1}\left(\dfrac{\dfrac{3}{4} + \dfrac{5}{12}}{1 - \dfrac{3}{4} \cdot \dfrac{5}{12}}\right)$

$= \tan^{-1}\left(\dfrac{\dfrac{36+20}{84}}{\dfrac{48-15}{84}}\right)$

$= \tan^{-1}\left(\dfrac{56}{33}\right)$

$=$ RHS

02.

$\sin^{-1}\left(\dfrac{3}{5}\right) + \cos^{-1}\left(\dfrac{12}{13}\right) = \sin^{-1}\left(\dfrac{56}{65}\right)$

STEP 1

$\sin^{-1}\left(\dfrac{3}{5}\right) = A$

$\dfrac{3}{5} = \sin A$

$\sin^2 A + \cos^2 A = 1$

$\dfrac{9}{25} + \cos^2 A = 1$

$\cos^2 A = 1 - \dfrac{9}{25}$

$\cos^2 A = \dfrac{16}{25}$

$\cos A = \dfrac{4}{5}$

$\tan A = \dfrac{\sin A}{\cos A}$

$\tan A = \dfrac{3}{4}$

$A = \tan^{-1}\left(\dfrac{3}{4}\right)$

HENCE $\sin^{-1}\left(\dfrac{3}{5}\right) = \tan^{-1}\left(\dfrac{3}{4}\right)$

STEP 2

$\cos^{-1}\left(\dfrac{12}{13}\right) = B$

$\dfrac{12}{13} = \cos B$

$\cos^2 B + \sin^2 B = 1$

$\dfrac{144}{169} + \sin^2 B = 1$

$\sin^2 B = 1 - \frac{144}{169}$

$\sin^2 B = \frac{25}{169}$

$\sin B = \frac{5}{13}$

$\tan B = \frac{\sin B}{\cos B}$

$\tan B = \frac{5}{12}$

$B = \tan^{-1}\left(\frac{5}{12}\right)$

HENCE $\cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\left(\frac{5}{12}\right)$

STEP 3

$\sin^{-1}\left(\frac{56}{65}\right) = C$

$\frac{56}{65} = \sin C$

$\sin^2 C + \cos^2 C = 1$

$\frac{56^2}{65^2} + \cos^2 C = 1$

$\cos^2 C = 1 - \frac{56^2}{65^2}$

$\cos^2 C = \frac{65^2 - 56^2}{65^2}$

$\cos^2 C = \frac{(65 + 56)(65 - 56)}{65^2}$

$\cos^2 C = \frac{(121)(9)}{65^2}$

$\cos C = \frac{11 x\ 3}{65}$

$\cos C = \frac{33}{65}$

$\tan C = \frac{\sin C}{\cos C}$

$\tan C = \frac{56}{33}$

$C = \tan^{-1}\left(\frac{56}{33}\right)$

HENCE $\sin^{-1}\left(\frac{56}{65}\right) = \tan^{-1}\left(\frac{56}{33}\right)$

STEP 4 :

$\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

WE PROVE

$\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{5}{12}\right) = \tan^{-1}\left(\frac{56}{33}\right)$

LHS

$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{5}{12}\right)$

$= \tan^{-1}\left(\dfrac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}}\right)$

$= \tan^{-1}\left(\dfrac{\frac{36+20}{84}}{\frac{48-15}{84}}\right)$

$= \tan^{-1}\left(\frac{56}{33}\right)$

$=$ RHS

03.
$$\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \sin^{-1}\left(\frac{77}{85}\right)$$

STEP 1

$$\sin^{-1}\left(\frac{3}{5}\right) = A$$

$$\frac{3}{5} = \sin A$$

$$\sin^2 A + \cos^2 A = 1$$

$$\frac{9}{25} + \cos^2 A = 1$$

$$\cos^2 A = 1 - \frac{9}{25}$$

$$\cos^2 A = \frac{16}{25}$$

$$\cos A = \frac{4}{5}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\tan A = \frac{3}{4}$$

$$A = \tan^{-1}\left(\frac{3}{4}\right)$$

HENCE $\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$

STEP 2

$$\sin^{-1}\left(\frac{8}{17}\right) = B$$

$$\frac{8}{17} = \sin B$$

$$\sin^2 B + \cos^2 B = 1$$

$$\frac{64}{289} + \cos^2 B = 1$$

$$\cos^2 B = 1 - \frac{64}{289}$$

$$\cos^2 B = \frac{225}{289}$$

$$\cos B = \frac{15}{17}$$

$$\tan B = \frac{\sin B}{\cos B}$$

$$\tan B = \frac{8}{15}$$

$$B = \tan^{-1}\left(\frac{8}{15}\right)$$

HENCE $\sin^{-1}\left(\frac{8}{17}\right) = \tan^{-1}\left(\frac{8}{15}\right)$

STEP 3

$$\sin^{-1}\left(\frac{77}{85}\right) = C$$

$$\frac{77}{85} = \sin C$$

$$\sin^2 C + \cos^2 C = 1$$

$$\frac{77^2}{85^2} + \cos^2 C = 1$$

$$\cos^2 C = 1 - \frac{77^2}{85^2}$$

$$\cos^2 C = \frac{85^2 - 77^2}{85^2}$$

$$\cos^2 C = \frac{(85 + 77)(85 - 77)}{85^2}$$

$$\cos^2 C = \frac{(162)(8)}{85^2}$$

$$\cos^2 C = \frac{(81)(16)}{85^2}$$

$$\cos C = \frac{9 \times 4}{85}$$

$$\cos C = \frac{36}{85}$$

$$\tan C = \frac{\sin C}{\cos C}$$

$$\tan C = \frac{77}{36}$$

$$C = \tan^{-1}\left(\frac{77}{36}\right)$$

HENCE $\sin^{-1}\left(\frac{77}{85}\right) = \tan^{-1}\left(\frac{77}{36}\right)$

STEP 4 :

$$\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \sin^{-1}\left(\frac{77}{85}\right)$$

WE PROVE

$$\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{8}{15}\right) = \tan^{-1}\left(\frac{77}{36}\right)$$

LHS

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{8}{15}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{4} + \frac{8}{15}}{1 - \frac{3}{4} \cdot \frac{8}{15}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{45+32}{60}}{\frac{60-24}{60}}\right)$$

$$= \tan^{-1}\left(\frac{77}{36}\right)$$

= RHS

04.
$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{5}{13}\right) = \cot^{-1}\left(\frac{119}{120}\right)$$

STEP 1

$$\cos^{-1}\left(\frac{12}{13}\right) = A$$

$$\frac{12}{13} = \cos A$$

$$\cos^2 A + \sin^2 A = 1$$

$$\frac{144}{169} + \sin^2 A = 1$$

$$\sin^2 A = 1 - \frac{144}{169}$$

$$\sin^2 A = \frac{25}{169}$$

$$\sin A = \frac{5}{13}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\tan A = \frac{5}{12}$$

$$A = \tan^{-1}\left(\frac{5}{12}\right)$$

HENCE $\cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\left(\frac{5}{12}\right)$

STEP 2

$$\sin^{-1}\left(\frac{5}{13}\right) = B$$

$$\frac{5}{13} = \sin B$$

$$\sin^2 B + \cos^2 B = 1$$

$$\frac{25}{169} + \cos^2 B = 1$$

$$\cos^2 B = 1 - \frac{25}{169}$$

$$\cos^2 B = \frac{144}{169}$$

$$\cos B = \frac{12}{13}$$

$$\tan B = \frac{\sin B}{\cos B}$$

$$\tan B = \frac{5}{12}$$

$$B = \tan^{-1}\left(\frac{5}{12}\right)$$

HENCE $\sin^{-1}\left(\frac{5}{13}\right) = \tan^{-1}\left(\frac{5}{12}\right)$

STEP 3 :

$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{5}{13}\right) = \cot^{-1}\left(\frac{119}{120}\right)$$

WE PROVE

$$\tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{5}{12}\right) = \tan^{-1}\left(\frac{120}{119}\right)$$

LHS

$$= \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{5}{12}\right)$$

$$= \tan^{-1}\left(\frac{\frac{5}{12} + \frac{5}{12}}{1 - \frac{5}{12} \cdot \frac{5}{12}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{60+60}{144}}{\frac{144-25}{144}}\right)$$

$$= \tan^{-1}\left(\frac{120}{119}\right)$$

$$= \text{RHS}$$

05.

$$2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$$

STEP 1

$$\sin^{-1}\left(\frac{3}{5}\right) = A$$

$$\frac{3}{5} = \sin A$$

$$\sin^2A + \cos^2A = 1$$

$$\frac{9}{25} + \cos^2A = 1$$

$$\cos^2A = 1 - \frac{9}{25}$$

$$\cos^2A = \frac{16}{25}$$

$$\cos A = \frac{4}{5}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\tan A = \frac{3}{4}$$

$$A = \tan^{-1}\left(\frac{3}{4}\right)$$

HENCE $\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$

STEP 2 :

$$2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$$

We Prove

$$2\tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$$

LHS

$$2\tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{4} + \frac{3}{4}}{1 - \frac{3}{4} \cdot \frac{3}{4}}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= \tan^{-1}\left(\frac{\frac{12+12}{16}}{\frac{16-9}{16}}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= \tan^{-1}\left(\frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24}{7} \cdot \frac{17}{31}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{744-119}{217}}{\frac{217+408}{217}}\right)$$

$$= \tan^{-1}\left(\frac{625}{625}\right)$$

$$= \tan^{-1}(1)$$

$$= \pi/4$$

06.

$$\sin^{-1}\left(\frac{3}{\sqrt{34}}\right) + \cos^{-1}\left(\frac{4}{\sqrt{17}}\right) = \frac{\pi}{4}$$

STEP 1

$$\sin^{-1}\left(\frac{3}{\sqrt{34}}\right) = A$$

$$\frac{3}{\sqrt{34}} = \sin A$$

$$\sin^2 A + \cos^2 A = 1$$

$$\frac{9}{34} + \cos^2 A = 1$$

$$\cos^2 A = 1 - \frac{9}{34}$$

$$\cos^2 A = \frac{25}{34}$$

$$\cos A = \frac{5}{\sqrt{34}}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\tan A = \frac{3}{5}$$

$$A = \tan^{-1}\left(\frac{3}{5}\right)$$

HENCE $\sin^{-1}\left(\frac{3}{\sqrt{34}}\right) = \tan^{-1}\left(\frac{3}{5}\right)$

STEP 2

$$\cos^{-1}\left(\frac{4}{\sqrt{17}}\right) = B$$

$$\frac{4}{\sqrt{17}} = \cos B$$

$$\cos^2 B + \sin^2 B = 1$$

$$\frac{16}{17} + \sin^2 B = 1$$

$$\sin^2 B = 1 - \frac{16}{17}$$

$$\sin^2 B = \frac{1}{17}$$

$$\sin B = \frac{1}{\sqrt{17}}$$

$$\tan B = \frac{\sin B}{\cos B}$$

$$\tan B = \frac{1}{4}$$

$$A = \tan^{-1}\left(\frac{1}{4}\right)$$

HENCE $\cos^{-1}\left(\frac{4}{\sqrt{17}}\right) = \tan^{-1}\left(\frac{1}{4}\right)$

STEP 3

$$\sin^{-1}\left(\frac{3}{\sqrt{34}}\right) + \cos^{-1}\left(\frac{4}{\sqrt{17}}\right) = \frac{\pi}{4}$$

We Prove

$$\tan^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{1}{4}\right) = \frac{\pi}{4}$$

$$= \tan^{-1}\left(\frac{\frac{3}{5} + \frac{1}{4}}{1 - \frac{3}{5} \cdot \frac{1}{4}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{12+5}{20}}{\frac{20-3}{20}}\right)$$

$$= \tan^{-1}\left(\frac{17}{17}\right)$$

$$= \tan^{-1}(1)$$

$$= \pi/4$$

07.
$$\sin^{-1}\left(\frac{2}{\sqrt{13}}\right) + \cos^{-1}\left(\frac{5}{\sqrt{26}}\right) = \frac{\pi}{4}$$

STEP 1

$$\sin^{-1}\left(\frac{2}{\sqrt{13}}\right) = A$$

$$\frac{2}{\sqrt{13}} = \sin A$$

$$\sin^2 A + \cos^2 A = 1$$

$$\frac{4}{13} + \cos^2 A = 1$$

$$\cos^2 A = 1 - \frac{4}{13}$$

$$\cos^2 A = \frac{9}{13}$$

$$\cos A = \frac{3}{\sqrt{13}}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\tan A = \frac{2}{3}$$

$$A = \tan^{-1}\left(\frac{2}{3}\right)$$

HENCE $\sin^{-1}\left(\frac{2}{\sqrt{13}}\right) = \tan^{-1}\left(\frac{2}{3}\right)$

STEP 2

$$\cos^{-1}\left(\frac{5}{\sqrt{26}}\right) = B$$

$$\frac{5}{\sqrt{26}} = \cos B$$

$$\cos^2 B + \sin^2 B = 1$$

$$\frac{25}{26} + \sin^2 B = 1$$

$$\sin^2 B = 1 - \frac{25}{26}$$

$$\sin^2 B = \frac{1}{26}$$

$$\sin B = \frac{1}{\sqrt{26}}$$

$$\tan B = \frac{\sin B}{\cos B}$$

$$\tan B = \frac{1}{5}$$

$$A = \tan^{-1}\left(\frac{1}{5}\right)$$

HENCE $\cos^{-1}\left(\frac{5}{\sqrt{26}}\right) = \tan^{-1}\left(\frac{1}{5}\right)$

STEP 3

$$\sin^{-1}\left(\frac{2}{\sqrt{13}}\right) + \cos^{-1}\left(\frac{5}{\sqrt{26}}\right) = \frac{\pi}{4}$$

We Prove

$$\tan^{-1}\left(\frac{2}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \frac{\pi}{4}$$

$$= \tan^{-1}\left(\frac{\frac{2}{3} + \frac{1}{5}}{1 - \frac{2}{3} \cdot \frac{1}{5}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{10+3}{15}}{\frac{15-2}{15}}\right)$$

$$= \tan^{-1}\left(\frac{13}{13}\right)$$

$$= \tan^{-1}(1)$$

$$= \pi/4$$

08. Prove

$$2\cot^{-1}\left(\frac{3}{2}\right) + \sec^{-1}\left(\frac{13}{12}\right) = \frac{\pi}{2}$$

STEP 1

$$\sec^{-1}\left(\frac{13}{12}\right) = A$$

$$\cos^{-1}\left(\frac{12}{13}\right) = A$$

$$\frac{12}{13} = \cos A$$

$$\cos^2 A + \sin^2 A = 1$$

$$\frac{144}{169} + \sin^2 A = 1$$

$$\sin^2 A = 1 - \frac{144}{169}$$

$$\sin^2 A = \frac{25}{169}$$

$$\sin A = \frac{5}{13}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\tan A = \frac{5}{12}$$

$$A = \tan^{-1}\left(\frac{5}{12}\right)$$

HENCE $\sec^{-1}\left(\frac{13}{12}\right) = \tan^{-1}\left(\frac{5}{12}\right)$

STEP 2

$$2\cot^{-1}\left(\frac{3}{2}\right) + \sec^{-1}\left(\frac{13}{12}\right)$$

$$= 2\tan^{-1}\left(\frac{2}{3}\right) + \tan^{-1}\left(\frac{5}{12}\right)$$

$$= \tan^{-1}\left(\frac{2}{3}\right) + \tan^{-1}\left(\frac{2}{3}\right) + \tan^{-1}\left(\frac{5}{12}\right)$$

$$= \tan^{-1}\left(\frac{\frac{2}{3} + \frac{2}{3}}{1 - \frac{2}{3} \cdot \frac{2}{3}}\right) + \tan^{-1}\left(\frac{5}{12}\right)$$

$$= \tan^{-1}\left(\frac{\frac{6+6}{9}}{\frac{9-4}{9}}\right) + \tan^{-1}\left(\frac{5}{12}\right)$$

$$= \tan^{-1}\left(\frac{12}{5}\right) + \tan^{-1}\left(\frac{5}{12}\right)$$

$$= \tan^{-1}\left(\frac{12}{5}\right) + \cot^{-1}\left(\frac{12}{5}\right)$$

$= \pi / 2$ …… $\tan^{-1}x + \cot^{-1}x = \pi / 2$

09. Evaluate

$$\sin\left(\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{5}{12}\right)\right)$$

STEP 1

$$\cos^{-1}\left(\frac{4}{5}\right) = A$$

$$\frac{4}{5} = \cos A$$

$$\cos^2 A + \sin^2 A = 1$$

$$\frac{16}{25} + \sin^2 A = 1$$

$$\sin^2 A = \frac{9}{25}$$

$$\sin A = \frac{3}{5}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\tan A = \frac{3}{4}$$

$$A = \tan^{-1}\left(\frac{3}{4}\right)$$

HENCE $\cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$

STEP 2 :

$$\sin\left(\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{5}{12}\right)\right)$$

$$= \sin\left(\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{5}{12}\right)\right)$$

$$= \sin \tan^{-1}\left(\frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}}\right)$$

$$= \sin \tan^{-1}\left(\frac{\frac{36 + 20}{48}}{\frac{48 - 15}{48}}\right)$$

$$= \sin \tan^{-1}\left(\frac{56}{33}\right)$$

STEP 3

$$\tan^{-1}\left(\frac{56}{33}\right) = B$$

$$\frac{56}{33} = \tan B$$

$$1 + \tan^2 B = \sec^2 B$$

$$1 + \frac{56^2}{33^2} = \sec^2 B$$

$$\sec^2 B = \frac{1089 + 3136}{33^2}$$

$$\sec^2 B = \frac{4225}{33^2}$$

$$\sec B = \frac{65}{33}$$

$$\cos B = \frac{33}{65}$$

$$\tan B = \frac{\sin B}{\cos B}$$

$$\frac{56}{33} = \frac{\sin B}{33/65}$$

$$\sin B = \frac{56}{65}$$

HENCE $\tan^{-1}\left(\frac{56}{33}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

FINALLY , THIS IS HOW THE SUM HAD PROGRESSED

$$\sin\left(\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{5}{12}\right)\right)$$

$$= \sin\left(\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{5}{12}\right)\right)$$

$$= \sin \tan^{-1}\left(\frac{56}{33}\right)$$

$$= \sin \sin^{-1}\left(\frac{56}{65}\right) = \frac{56}{65}$$

SOLUTION TO Q SET - 3

01. $\tan^{-1}\left(\dfrac{\sin 2x}{1+\cos 2x}\right) = x$

$= \tan^{-1}\left(\dfrac{2\sin x \cdot \cos x}{2\cos^2 x}\right)$

$= \tan^{-1}\left(\dfrac{\sin x}{\cos x}\right)$

$= \tan^{-1}[\tan x]$

$= x \qquad = \text{RHS}$

02. $\cot^{-1}\left(\dfrac{\sin 2x}{1-\cos 2x}\right) = x$

$= \cot^{-1}\left(\dfrac{2\sin x \cdot \cos x}{2\sin^2 x}\right)$

$= \cot^{-1}\left(\dfrac{\cos x}{\sin x}\right)$

$= \cot^{-1}[\cot x]$

$= x \qquad = \text{RHS}$

03. $\tan^{-1}\sqrt{\dfrac{1-\cos 2x}{1+\cos 2x}} = x$

$= \tan^{-1}\sqrt{\dfrac{2\sin^2 x}{2\cos^2 x}}$

$= \tan^{-1}\dfrac{\sin x}{\cos x}$

$= \tan^{-1}[\tan x]$

$= x \qquad = \text{RHS}$

04. $\tan^{-1}[\operatorname{cosec} x - \cot x] = x/2$

$= \tan^{-1}\left(\dfrac{1-\cos x}{\sin x}\right)$

$= \tan^{-1}\left(\dfrac{2\sin^2 x/2}{2\sin x/2 \cdot \cos x/2}\right)$

$= \tan^{-1}\left(\dfrac{\sin x/2}{\cos x/2}\right)$

$= \tan^{-1}[\tan x/2]$

$= x/2 \qquad = \text{RHS}$

05. $\tan^{-1}\sqrt{\dfrac{1-\cos x}{1+\cos x}} = x/2$

$= \tan^{-1}\sqrt{\dfrac{2\sin^2 x/2}{2\cos^2 x/2}}$

$= \tan^{-1}\left(\dfrac{\sin x/2}{\cos x/2}\right)$

$= \tan^{-1}[\tan x/2]$

$= x/2 \qquad = \text{RHS}$

06. $\tan^{-1}\left(\dfrac{\cos x - \sin x}{\cos x + \sin x}\right) = \pi/4 - x$

$= \tan^{-1}\left(\dfrac{\dfrac{\cos x - \sin x}{\cos x}}{\dfrac{\cos x + \sin x}{\cos x}}\right)$

$$= \tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$$

$$= \tan^{-1}\left(\frac{\tan \pi/4 - \tan x}{1+\tan \pi/4 \cdot \tan x}\right)$$

$$= \tan^{-1} \tan(\pi/4 - x)$$

$$= (\pi/4 - x) \qquad = \text{RHS}$$

07. $\cot^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}} = \pi/4 + x/2$

$$= \tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$$

$$= \tan^{-1}\sqrt{\frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}}$$

$$= \tan^{-1}\sqrt{\frac{(\cos x/2 + \sin x/2)^2}{(\cos x/2 - \sin x/2)^2}}$$

$$= \tan^{-1}\left(\frac{\cos x/2 + \sin x/2}{\cos x/2 - \sin x/2}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\cos x/2 + \sin x/2}{\cos x/2}}{\frac{\cos x/2 - \sin x/2}{\cos x/2}}\right)$$

$$= \tan^{-1}\left(\frac{1 + \tan x/2}{1 - \tan x/2.}\right)$$

$$= \tan^{-1}\left(\frac{\tan \pi/4 + \tan x/2}{1 - \tan \pi/4 . \tan x/2.}\right)$$

$$= \tan^{-1} \tan(\pi/4 + x/2)$$

$$= \pi/4 + x/2 \qquad = \text{RHS}$$

08. $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) = \pi/4 + x/2$

$$= \tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\frac{(\cos x/2 - \sin x/2)(\cos x/2 + \sin x/2)}{(\cos x/2 - \sin x/2)^2}\right)$$

$$= \tan^{-1}\left(\frac{\cos x/2 + \sin x/2}{\cos x/2 - \sin x/2}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\cos x/2 + \sin x/2}{\cos x/2}}{\frac{\cos x/2 - \sin x/2}{\cos x/2}}\right)$$

$$= \tan^{-1}\left(\frac{1 + \tan x/2}{1 - \tan x/2.}\right)$$

$$= \tan^{-1}\left(\frac{\tan \pi/4 + \tan x/2}{1 - \tan \pi/4 . \tan x/2.}\right)$$

$$= \tan^{-1} \tan(\pi/4 + x/2)$$

$$= \pi/4 + x/2 \qquad = \text{RHS}$$

09. $\tan^{-1}\left(\dfrac{a\cos x - b\sin x}{b\cos x + a\sin x}\right) = x$

$= \tan^{-1}\left(\dfrac{\dfrac{a\cos x - b\sin x}{b\cos x}}{\dfrac{b\cos x + a\sin x}{b\cos x}}\right)$

$= \tan^{-1}\left(\dfrac{\dfrac{a\cos x}{b\cos x} - \dfrac{b\sin x}{b\cos x}}{\dfrac{b\cos x}{b\cos x} + \dfrac{a\sin x}{b\cos x}}\right)$

$= \tan^{-1}\left(\dfrac{\dfrac{a}{b} - \tan x}{1 + \dfrac{a}{b}.\tan x}\right)$

$= \tan^{-1}\dfrac{a}{b} - \tan^{-1}\tan x$

$= \tan^{-1}\dfrac{a}{b} - x$

SOLUTION TO Q SET - 4

01. $\cos^{-1}(4x^3 - 3x) = 3\cos^{-1}x$

LHS

$= \cos^{-1}(4x^3 - 3x)$

Put $x = \cos\theta$

$= \cos^{-1}(4\cos^3\theta - 3\cos\theta)$

$= \cos^{-1}(\cos 3\theta)$

$= 3\theta$

$= 3\cos^{-1}x \quad = \text{RHS}$

02. $\cos{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right) = 2\tan^{-1}x$

LHS

$= \cos{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right)$

Put $x = \tan\theta$

$= \cos^{-1}\left(\dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}\right)$

$= \cos^{-1}\cos 2\theta$

$= 2\theta$

$= 2\tan^{-1}x \quad = \text{RHS}$

03. $\sin{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right) = \dfrac{\pi}{2} - 2\tan^{-1}x$

LHS

$= \sin^{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right)$

Put $x = \tan\theta$

$= \cos^{-1}\left(\dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}\right)$

$= \sin^{-1}\cos 2\theta$

$= \sin^{-1}\sin(\pi/2 - 2\theta)$

$= \pi/2 - 2\theta$

$= \pi/2 - 2\tan^{-1}x \quad = \text{RHS}$

04. $\tan^{-1}\left(\dfrac{3x - x^3}{1 + 3x^2}\right) = 3\tan^{-1}x$

LHS

$= \tan^{-1}\left(\dfrac{3x - x^3}{1 + 3x^2}\right)$

Put $x = \tan\theta$

$= \tan^{-1}\left(\dfrac{3\tan\theta - \tan^3\theta}{1 + 3\tan^2\theta}\right)$

$= \tan^{-1} \tan 3\theta$

$= 3\theta$

$= 3\tan^{-1} x \qquad = \text{RHS}$

05. $\tan^{-1}\left(\dfrac{x}{\sqrt{a^2 - x^2}}\right) = \sin^{-1}\left(\dfrac{x}{a}\right)$

LHS

$= \tan^{-1}\left(\dfrac{x}{\sqrt{a^2 - x^2}}\right)$

Put $x = a\sin\theta$

$= \tan^{-1}\left(\dfrac{a\sin\theta}{\sqrt{a^2 - a^2\sin^2\theta}}\right)$

$= \tan^{-1}\left(\dfrac{a\sin\theta}{\sqrt{a^2(1 - \sin^2\theta)}}\right)$

$= \tan^{-1}\left(\dfrac{a\sin\theta}{\sqrt{a^2\cos^2\theta}}\right)$

$= \tan^{-1}\left(\dfrac{a\sin\theta}{a\cos\theta}\right)$

$= \tan^{-1} \tan\theta$

$= \theta$ →

$x = a \cdot \sin\theta$
$\text{Sin}\,\theta = \dfrac{x}{a}$
$\theta = \sin^{-1}\dfrac{x}{a}$

$= \sin^{-1}\left(\dfrac{x}{a}\right)$

$=$ RHS

06. $\tan^{-1}\sqrt{\dfrac{1 - x}{1 + x}} = \dfrac{1}{2}\cos^{-1}x$

LHS

$= \tan^{-1}\sqrt{\dfrac{1 - x}{1 + x}}$

Put $x = \cos\theta$

$= \tan^{-1}\sqrt{\dfrac{1 - \cos\theta}{1 + \cos\theta}}$

$= \tan^{-1}\sqrt{\dfrac{2\sin^2\theta/2}{2\cos^2\theta/2}}$

$= \tan^{-1}\sqrt{\dfrac{\sin{}^{\theta}/_{2}}{\cos{}^{\theta}/_{2}}}$

$= \tan^{-1} \tan{}^{\theta}/_{2}$

$= {}^{\theta}/_{2}$

$= \dfrac{1}{2}\cos^{-1}x$

07.
$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}(x^2)$$

LHS : Dividing numerator & denominator by $\sqrt{1+x^2}$

$$= \tan^{-1}\left(\frac{1+\sqrt{\frac{1-x^2}{1+x^2}}}{1-\sqrt{\frac{1-x^2}{1+x^2}}}\right)$$

$$= \tan^{-1}(1) + \tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}$$

$$= \frac{\pi}{4} + \tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}$$

Put $x^2 = \cos 2\theta$

$$= \frac{\pi}{4} + \tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$$

$$= \frac{\pi}{4} + \tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}$$

$$= \frac{\pi}{4} + \tan^{-1}(\tan\theta)$$

$$= \frac{\pi}{4} + \theta \quad ;$$

$$= \frac{\pi}{4} + \frac{1}{2}\cos^{-1}(x^2)$$

NOW

$\cos 2\theta = x^2$

$2\theta = \cos^{-1}(x^2)$

$\theta = \frac{1}{2}\cos^{-1}(x^2)$

EXTRA

$$\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$$

NOTE

if $xy > 1$ THEN

$$\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

LHS

$$= \tan^{-1}1 + \pi + \tan^{-1}\left(\frac{2+3}{1-2.3}\right)$$

$$= \tan^{-1}1 + \pi + \tan^{-1}\left(\frac{5}{-5}\right)$$

$$= \tan^{-1}1 + \pi + \tan^{-1}(-1)$$

$$= \tan^{-1}1 + \pi - \tan^{-1}1$$

$$= \pi$$